THE CASE OF THE NICE, COLD BEER

The Bungle Brewery
1729 Wort Way
East Yeastville, PU 11235
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Pre-Calculus Student
Franklin & Marshall College
Lancaster, PA 17604

Dear Pre-Calculus Student:

I have a difficult problem for which no one in my brewery can find an easy solution, and I hope that you can help me. Your intrepid and enterprising professor, Dr. Culp-Ressler, referred me to you.

At the Bungle Brewery, we make 42 different flavors of beer; although we make very little of any one kind of beer—each flavor is a "limited edition". Because of this specialization, quality is extremely important to us. Unfortunately, lately we've been having troubles with quality control because of difficulties in timing the cooling of the wort (pronounced "wirt"). This is why we've come to you.

As you may know from listening to your esteemed professor in class, the preparation process is this: we mix large cans of malt extract with water to form the wort and then we boil the wort for an hour.

The next stage is to remove the wort from the burner and cool the mixture by placing it in a large tub of ice-water, which is kept near the freezing point by our refrigerating mechanisms. The wort is cooled from boiling to room-temperature—70°F Fahrenheit—and then the yeast is added to ferment the mixture. If the wort is much warmer than that, the yeast dies quickly, and the whole mixture is ruined. If the wort gets much colder than room temperature, the yeast works too slowly, and the beer will not measure up to our high standards.

It's at this cooling stage that we have problems. We have very
few employees, who have lots of tasks to accomplish. One of our new employees, in charge of looking after the Bungle Blueberry Brew, let the mixture cool for an hour. When he came back, the temperature had dropped about 80 degrees, down to 131° Fahrenheit. He measured the temperature again after another hour, and found it was now 86° F, still too hot for the yeast. Alas, he was called away to work on the Bungle Black Bean Brew, and when he returned one hour later the mixture had cooled to 62° F—too cold for the yeast. Our Bungle Blueberry Brew was ruined.

We need to find a way to determine when the mixture will be at the right temperature, so we don't make similar mistakes again. Running on a shoestring budget such as ours doesn't leave us much room for experimentation, so instead we're hoping you can help us out mathematically. We've done a little library search, and came up with something we think can help you. In a book called Discovering Calculus by Levine & Rosenstein (McGraw-Hill, 1994), we found a formula on page 5 - 43. This formula says that the temperature of a cooling object can be determined by using the formula

\[ T(t) = b + (T_0 - b)e^{-kt}. \]

All these variables are pretty confusing; I've seen that letter \( e \) before and it just confuses me; and I have no idea what \( k \) is or how to find it. The best I can make out, \( b \) is the temperature of the ice-water, \( T_0 \) is the temperature of the wort when it's boiling (about 212° F), and \( T(t) \) is the temperature of the wort \( t \) hours after it's removed from boil. I think someone who knows math could figure \( k \) out from the measurements we've already made; what I really need to know is when the temperature of the wort will reach 70° F.

The question I have for you is: Given the procedure and the measurements described above, can you tell us how long we should leave the wort in the ice-bath? I would appreciate an answer as soon as possible, but hopefully no later than October 21 (our budget is due the day after).

Yours sincerely,
P.S. A member of our Public Relations Department just urged me to ask you also for suggestions for new flavors, and to remind you that we have "bathtub" discounts on volume sales.

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