Dear Tasha,

We decided to write this letter directly to you because from our experiences with Al, we know that he is not only unreliable, but also extremely careless. So to avoid letting him put all of our work to waste, we passed him up as the middleman by simply cutting him out of the loop. We suggest to you not to take any further problems to him; he is very incompetent.

Anyway, you had an issue that you needed resolved, and being Dr. Crannell's Calculus II loving students, we accepted your plea for help. You asked us to help calculate the minimum floor height for a loft over your extremely oddly shaped dining room required to meet the San Diego building standards code. Assuming that you do live in San Diego, as well as that your floor, ceiling and walls are all straight and level and that there are no dips or curves in them, and finally that the thickness of the loft floor is negligible, we found the appropriate height of the entire room should be 17.47 feet. This means that the distance between the floor of the dining room and the floor of the loft, or the total height of the dining room will have a height of 17.47 "endash; 7.5 = 9.97ft.

It is our understanding that you have previously hired an architect to measure the height of this room as well. The architect you consulted would have come very close to the appropriate average ceiling height with his method of measuring from the middle of the room. Using this method, he found the average height to be about 17.42 feet, meaning that the total height of the dining room would only be 17.42 "endash; 7.5 = 9.92ft. With the help of Dr. Crannell, we assumed that the architect's method would yield the same result as averaging the height of the height of wall 1 and wall 2 as labeled below. Because we were not physically present and able to use the tape measure to take the measurements for ourselves, we solved for the equivalent length of the tape measure mathematically.

We found this average height by adding the height of the two walls together,

\[
18 \frac{3}{12} \text{ ft} + 16 \frac{3}{12} \text{ ft} = 34 \frac{10}{12} \text{ ft}.
\]

Next we divided this number by two to get the final answer,
This 17.417 feet would be the height of the whole room from the floor to the ceiling of the dining room.

Now, you asked us to check and make sure that the height that the architect got was appropriate. Our method differed from his in that we used our knowledge that the average height of the room is equal to total volume of the room divided by the area of the base (floor) of the room, so

\[ \text{Average Height (ft)} = \frac{\text{Volume (ft}^3\text{)}}{\text{Base Area (ft}^2\text{)}}. \]

So, we started by finding the area of the base of the room using the picture of the floor that you supplied us with. (Throughout this entire paper you can assume that the words base length and height refer to specific sides of shapes within your dining room that we are discussing at that moment.)

To mathematicians like us, your dining room floor can be seen as two separate shapes, a triangle and a rectangle:

We know that the area of a rectangle is equal to the length multiplied by the height, so the area is

\[ 9.5 \times 10 = 95 \text{ ft}^2. \]
The area of a triangle is equal to half the length multiplied by the height, or

\[(9.5 \times 5)/2 = 23.75 \text{ ft}^2.\]

To find the total area of the base of the room (floor), we added the area of the triangle part of your floor and the rectangle part of your floor to get

\[.95 + 23.75 = 118.75 \text{ ft}^2.\]

The next step to our process was to find the total volume of the room. Once again, we needed to break the room up into parts. Ultimately, we had four parts. A three dimensional view of your dining room is given below for clarification.

The four shapes that we split your dining room into will be designated with colors. A rectangular one will be pink, a triangular one will be blue, another triangular one will be yellow, and a pyramid will be orange. We started with the rectangular shape (pink).

For a shape like this, the volume is equal to the base times the length times the height, so
The next shape we measured was one of the triangular ones (blue).

\[
10 \cdot \frac{9}{2} \cdot 16 \frac{7}{12} = \frac{37810}{24} \text{ ft}^3.
\]

The next shape we measured was the other triangular shape (yellow).

The volume of a shape like this is equal to one half of the base times the length times the height, so

\[
\frac{(10 \cdot \frac{9}{2} \cdot \frac{19}{12})}{2} = \frac{1900}{24} \text{ ft}^3.
\]

The volume of this shape is found in the same way as it was for the last shape, one half of the base times the height times the length:

\[
\frac{\frac{9}{2} \cdot 16 \frac{7}{12} \cdot 5}{2} = \frac{18905}{48} \text{ ft}^3.
\]
The third shape we measured was the pyramid, and you should know this shape provided us with the most difficulty.

Despite the difficulty, with the help of Dr. Crannell, we finally realized that the volume of this orange shape is equal to one third of the base times the length times the height, or

\[
\frac{\left(5 \times \frac{9}{2} \times 9 \frac{1}{2}\right)}{3} = \frac{1900}{72} \text{ ft}^3.
\]

Once we found the volume of each individual shape of the room, only one task remained. We needed to add all of the volumes together, logically, to get the total volume of your dining room, therefore

\[
\frac{378.10}{24} + \frac{1900}{24} + \frac{1890.5}{48} + \frac{1900}{72} = 2074.826 \text{ ft}^3.
\]

As we noted earlier, the average height can be found by dividing the base area (area of the floor) into the total volume of the room. So this is what we did, so that the total average height of the whole room, pre-loft was 17.47 feet:

\[
\frac{2074.826 \text{ ft}^3}{118.75 \text{ ft}^3} = 17.47 \text{ ft.}
\]

Once you remove the 7.5 feet for the height of the loft, the floor of your loft should ideally be 17.47 \&endash; 7.5 = 9.97 feet above the floor of your dining room. The architect's method was only off by about a mere of an inch, but who knows, maybe one Thanksgiving you'll be glad you had all that extra space in your dining room!

Sincerely,

The Tricky Trio, \quad \textit{Sarah, Parker, and Pete}