September 26, 2003

From: Christy Strunk (Calc I Student)
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Dear Myron,

Do not worry any longer; I received your letter, and I can help you with your financial dilemma. You surely are lucky to have had a friend like Mr. Gusterson. The amount of money he left you was very generous, and I can tell you just how much that is, along with how much he left to Philomena. Your portion of the check totals $3,456.77, and $7,949.37 goes to your love. Hopefully with my help and Mr. Gusterson's kind gift you will be able to win over Philomena.

First I want to clear up a few misconceptions you may have had concerning the interest. When you wrote the note to yourself after the first year the money was in the bank, you must have assumed that the amount of interest earned each year would be the same. With this in mind you took the $438.30 that you earned the first year and multiplied it by 10, thinking that this would give you the total amount of interest that would be earned over the 10 years. However, the interest rate is multiplied by the total amount of money in the bank at that time (including the amount earned from the previous year's interest), thus you will earn more interest money each year. Also, when calculating the portions of money belonging to each of you, we are going to make a few assumptions. We're assuming that there were no additional fees or any other sort of charges associated with the account, and that the check you received is just the initial deposit with added interest earnings.

With this in mind, the first step to figuring out how much money belongs to you and how much goes to your dearest Philomena is to use the equation typically associated with fixed interest rates, which is

\[ A(t) = A_0(1 + r)^t \]

In this formula, \( A(t) \) represents the amount of money in the CD after \( t \) years. \( A_0 \) is the original amount of money put in, and \( r \) is the interest rate.

Knowing these variables, we can insert some of your known numbers to make an equation. In this equation $11,406.14 will be used for \( A(t) \), and 10 will be used for \( t \), meaning after ten years the total amount of money in the CD is $11,406.14. The equation will read as follows:
Some other information to take into consideration is the note from the statement you received after the first year. To reach the total of $438.30 of interest earned in the first year, the starting amount of the deposit would have to have been multiplied by the interest rate. Using the same variables as above, the equation would be

$$438.30 = A_0r.$$  

Here you might notice that both of the equations have the same unknown variables. We can use a method called substitution to combine these two equations to make one that can then be solved for. It will be easiest to solve the second equation for $A_0$, and use this as the substituting equation. After substituting and simplifying, I found the following equation:

$$26.02 = 1/r (1 + r)^{10}.$$  

At this point we are basically trying to find the interest rate ($r$) for the CD, and we will use this information to determine the original amount. A graph is the most efficient way to find the value of $r$. So assuming $r$ is the independent variable, and placing it on the horizontal axis of the graph, we will draw a line across the graph at 26.02 on the vertical axis. The $r$ value where the figure intersects this line will be the interest rate.

[figure deleted]

As you can see on the graph, the intersection point is at $r = .127$. Since interest rate is in percent we multiply this number by 100, therefore the interest rate is 12.7%. You might notice that there is another point of intersection. Do not let this confuse you. By testing out this first point in the equation we can see that it does not work, and is not the point we are interested in for this problem, so we are going to disregard it.

Now to get to the heart of your problem and put all these numbers into the specific values you're looking for, we will plug the newly found $r$ value into our equation from above and solve for $A_0$, which is the initial amount. The quotient comes out to be $3,451.18$. You might notice that this number is slightly different from the amount that I said belonged to you, you'll see why next.

Just to check our numbers and make sure that Philomena gets what she's entitled to, I solved the equation again, using the initial amount that was just determined. However, it turns out that this number is just a little off. Don't worry though, because a little adjustment showed that by using 12.68% as the interest rate, we get $3,456.77 as the initial amount, and this checks out much better.
Since you are to keep the initial deposit only, you should keep $3,456.77 of the check. By subtracting this number from the total we get Philomena's portion, which is $7,949.37.

I wish you the best of luck with Philomena. I have a feeling that Mr. Gusterson was right and that things will turn out for the best.

Sincerely,

Christy

P.S. I would like to say thank you to several people for making the accuracy of this letter possible, including Dr. Crannell and Sarah Hammel. The Texas Instruments company is appreciated for invention of the TI-83 graphing calculator. Also thanks to Dana Tambellini for assisting in fixing my printer so that this letter could be sent.

P.P.S. Sorry I cannot be of assistance in the rhyming, my creativity just isn't what it used to be.