Dear Myron,

First of all, your poem was really not that good. How are you going to win over a real woman with something like that? Here's a suggestion of something you might want to try:

\[
\text{Roses are red,} \\
\text{Candies are sweet,} \\
\text{If you're a vegetarian,} \\
\text{I won't make you eat meat.}
\]

Doesn't that have a better ring to it? But, that wasn't the only reason you wrote, was it?

If we understand your other problem correctly, these are the facts: you are looking to make some storage boxes for Victor that can be stacked and hold hardware equipment, while the front of the box has a hole for customers to reach in and get what they want. They are 12 inches deep, but you need to know how high and wide they need to be. Also, the bottom part of the box holds 486 cubic inches. The boxes are made of iron sheeting that costs \$0.01 per square inch, and soldering that costs \$0.18 per square inch. You of course are looking for the cheapest way to make these boxes. Victor says that they should be 6 inches wide and 12 inches high. You are essentially asking what the width and height of the box are supposed to be in order to minimize the cost.

We are going to assume that these boxes will be produced exactly in the shape necessary and not in a rectangular format, basically that you are paying only for the metal that goes into making the box. We will also presume that no materials or specifications are necessary in order to make them safer for the customers. It would be a shame if potentially rough metal started cutting up customers, making them bleed all over the supplies. Essentially, the following calculations are done based entirely upon the information you have given us and nothing else.

To answer your question, the best dimensions of the storage boxes are 9 inches wide, 10.5 inches high, and 12 inches deep, yielding a minimum cost of \$15.21 per box.

A diagram labeling the values of the box uses \(w\) to represent width, and \(h\) to represent the height of the front flap. It looks like this.
Firstly, it is necessary to determine a relationship between the height and width of the boxes. This is done by determining a formula for the volume, a value that is equal to the length, width, and height multiplied together (a fact that every 3rd grader knows). Knowing that the bottom section of the box that will hold the materials when placed in it has a volume of 486 in$^3$, it is possible then to derive an equation for the volume of the box. The volume of the bottom of the box added to the volume of the top of the box, yielding the total volume, is described by the formula

$$486 + 72w = V = 12w(h + 6) ,$$

where $w$ represents width in inches, $V$ signifies volume in cubic inches, and $h$ corresponds to the height of the front of the box in inches. The above equation can be reduced to be

$$w = \frac{40.5}{h} .$$

There are two equations that are significant in solving this problem, in that they both make up the price that we are trying to minimize. The first is the cost of the material to build the box, knowing that the iron sheeting is worth $0.01$ per square inch. Adding up the areas of each face of the cube and then multiplying it all by the cost of the material yields the total cost to produce the material of one box. This is expressed according to the equation

$$\text{cost sheet metal} = .01(12w + 12w + (h + 6)w + hw + 12(h + 6) + 12(h + 6) ,$$

which can be reduced to the equation

$$\text{cost sheet metal} = .02hw + 0.3w + 0.24h + 1.44 .$$

The second equation is the cost of the soldering that holds the edges of the box together, which costs $0.18$ per inch. Because only certain edges of the box will be soldered, not all of the edges are involved in the equation for the total cost of soldering, as seen in Figure 1 where the parts to be soldered are highlighted in pink. The cost of soldering a box is determined according to the equation
costsoldering = 0.18(12 + w + (h + 6) + (h + 6) + h + h) ,

which can be reduced to the equation

costsoldering = 0.18w + .72h + 4.32 .

The two above equations are added to describe the total cost of making a single box. First however, the equation

\[ w = \frac{40.5}{h} \]

needs to be substituted into each of the equations describing cost in order to have a single variable, h, when forming a single formula. The formula for sheet metal cost becomes

\[ \text{costsheet metal} = \frac{12.15}{h} + .24h + 2.25 , \]

and the cost of soldering is equal to the formula

\[ \text{costsoldering} = \frac{7.29}{h} + .72h + 4.32 . \]

The sum of the above equations for soldering and sheet metal represents the total cost of making a single box, making the equation

\[ \text{costtotal} = \left( \frac{12.15}{h} + .24h + 2.25 \right) + \left( \frac{7.29}{h} + .72h + 4.32 \right) , \]

which, when simplified, yields the equation

\[ \text{costtotal} = 19.44/h + .96h + 6.57 . \]

At this point, a graph can be drawn for total cost, and the minimum cost can be found by means of a graphing calculator.

[figure deleted]

FIGURE 2. Cost vs. Height
To find the minimum of the graph of total cost algebraically, this is equivalent to determining the point on the graph at which the tangent line is simply a horizontal line with no slope, namely, the derivative. The first derivative of the total cost is

\[(\text{cost}_{\text{total}})' = -19.44h - 2 + .96\]

which is equal to zero at the points 4.5 and &endash;4.5. The negative value is discarded because it is impossible to have a negative length.

The determined h value can now be used to calculate all of the other dimensions. Using the initial formula relating height and width, width is determined according to the equation

\[w = 40.5/(4.5)\]

where w represents width, and is simplified to be

\[w = 9\, .\]

The best dimensions of the box can now be shown according to the following picture.

[figure deleted]

FIGURE 3. Best Dimensions (with areas) in Plan View of Unassembled Box

Using the determined value for height, the equation determining total cost can be filled in and solved. Therefore, the amount of money required to make one box is found with the equation

\[\text{cost}_{\text{total}} = 19.44/h + .96h + 6.57\, ,\]

which when substituting 4.5 for h, the height, yields a total cost of $15.21. However, if you would like to substitute in Victor's values of 12 inches high and 6 inches wide, a higher cost per box will result. Substituting his values for width and height into the equations for the cost of sheet metal and cost of soldering, and then summing the result, yields the total cost according to Victor. This is demonstrated according to the following formulas (previously presented) with the appropriate substitutions:
costsheet metal = .01(12(6) + 12(6) + (12 + 6)(6) + 12(6) + 12(12 + 6) + 12(12 + 6))

and

costsoldering = 0.18(12 + 6 + (12 + 6) + (12 + 6) + 12 + 12) .

Simplifying the results, the cost of sheet metal is reduced to $7.56, and the cost of soldering is simplified to $14.04. Summing the cost of sheet metal and cost of soldering, Victor's dimensions yield a total cost of $21.60 to make one box. Note that the difference in cost between his value and our value is $6.39 per box.

Well, I hope we answered your question without too much confusion. Oh, and by the way, we thought of another poem that might suit your needs. It goes like this:

> If you get to know me, Philomena,
>     You'll think I'm very nice.
>     You'll also soon find out,
>     I don't have any lice.
>     So I ask you now:
>     Won't you please be my bride?
>     And we can owe all our happiness,
>             To that nice geezer that died.

Sincerely,

*Amos, Timothy, and Alexander*

P.S. We think you should give a special thank you to Dr. Crannell for all of her wonderful advice which she so willingly gave to us regarding the assumptions, mathematical procedure, and overall infinite wisdom.

P.P.S. You should name your kids "Harry" and "Lloyd."